

# A SIAC filtering toolbox for Finite Volume & Finite Element schemes



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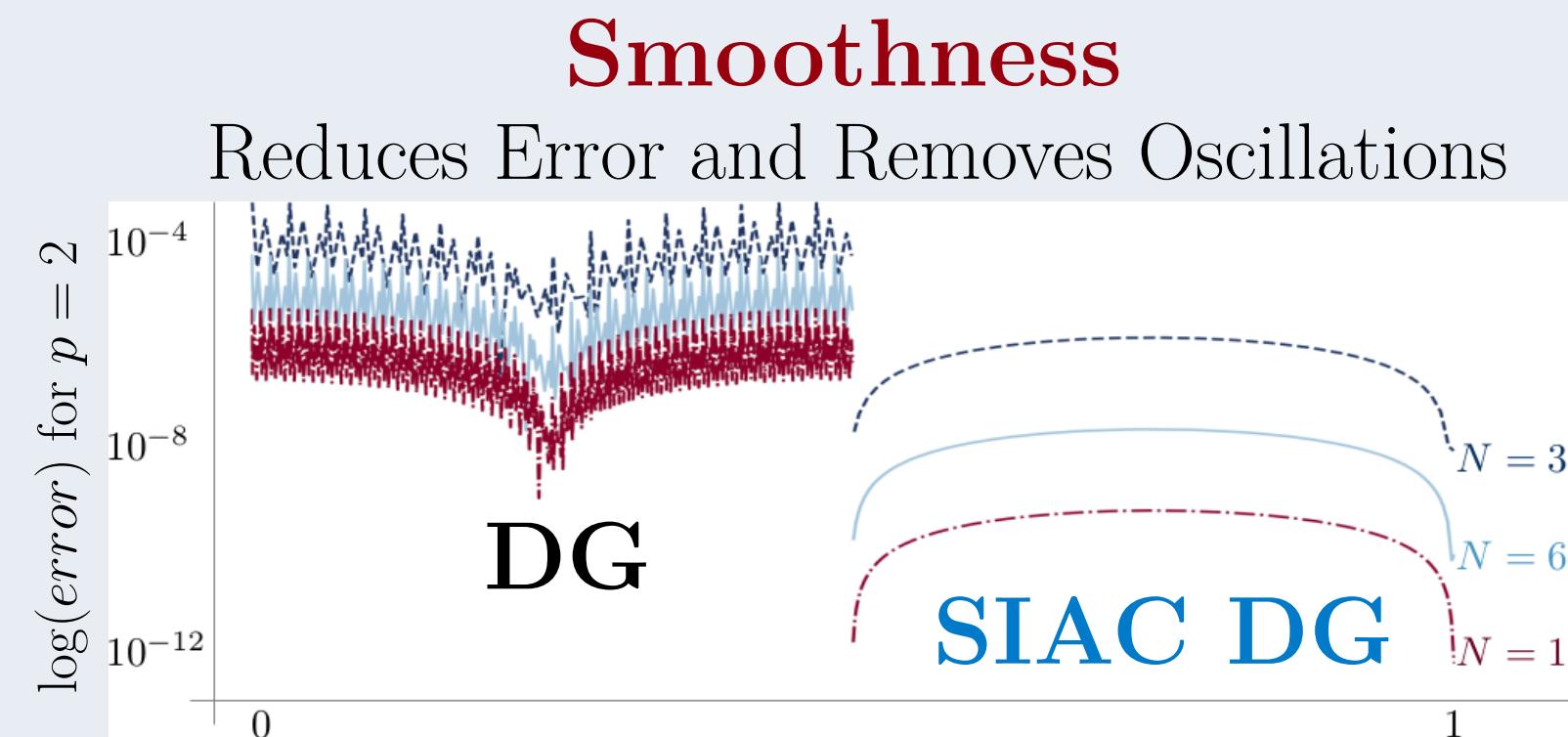
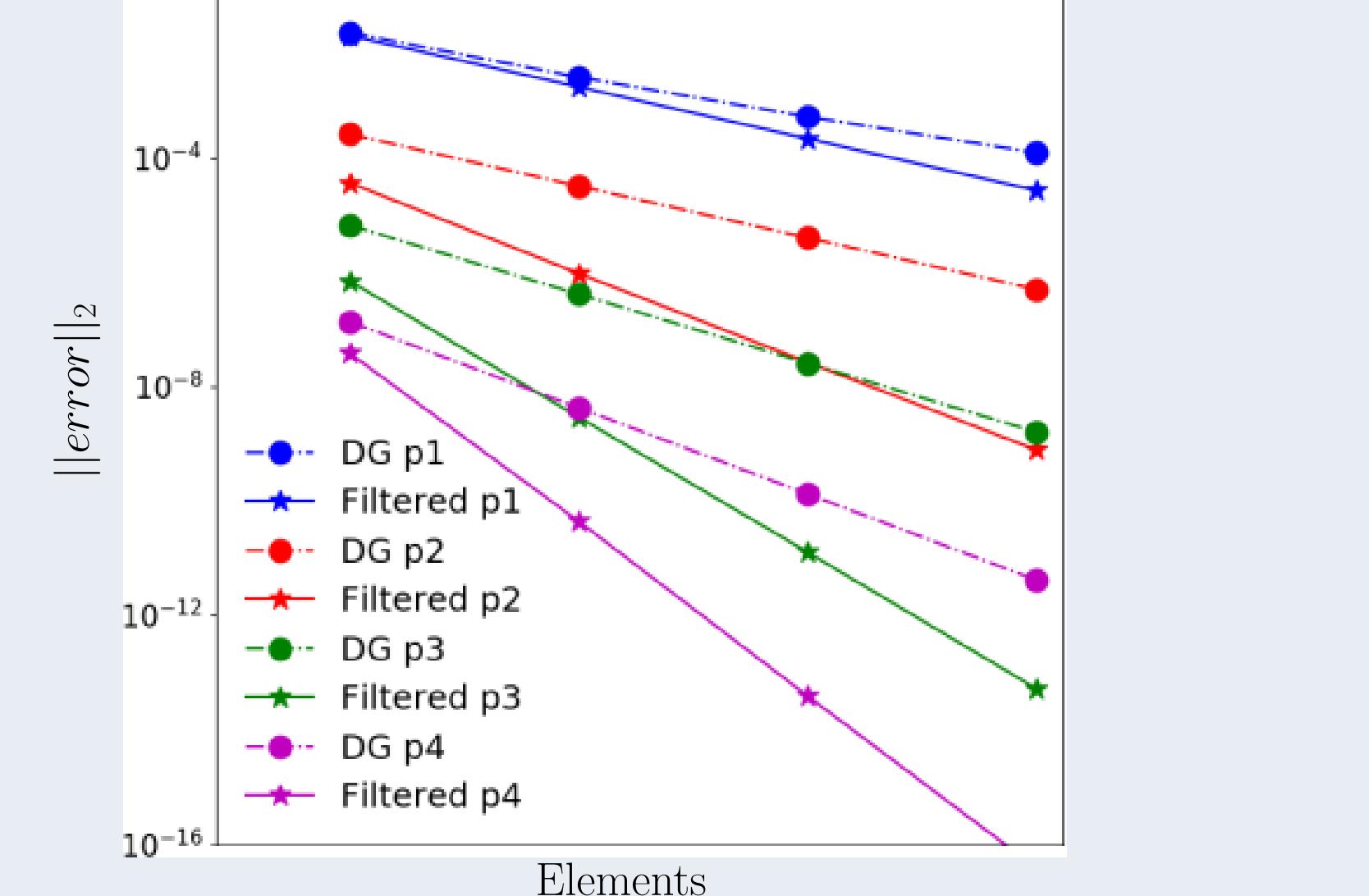


## Why use a SIAC Filter?

The Smoothness-Increasing Accuracy-Conserving (SIAC) filter achieves:

**Superconvergence**  
(order) DG  $\rightarrow p+1$  | SIAC DG  $\rightarrow 2p+1$

$$u^*(\bar{x}, \bar{y}) = \frac{1}{h_x h_y} \int_{\mathbb{R}} \int k_x(\bar{x}-x) k_y(\bar{y}-y) u(x, y) dy dx$$



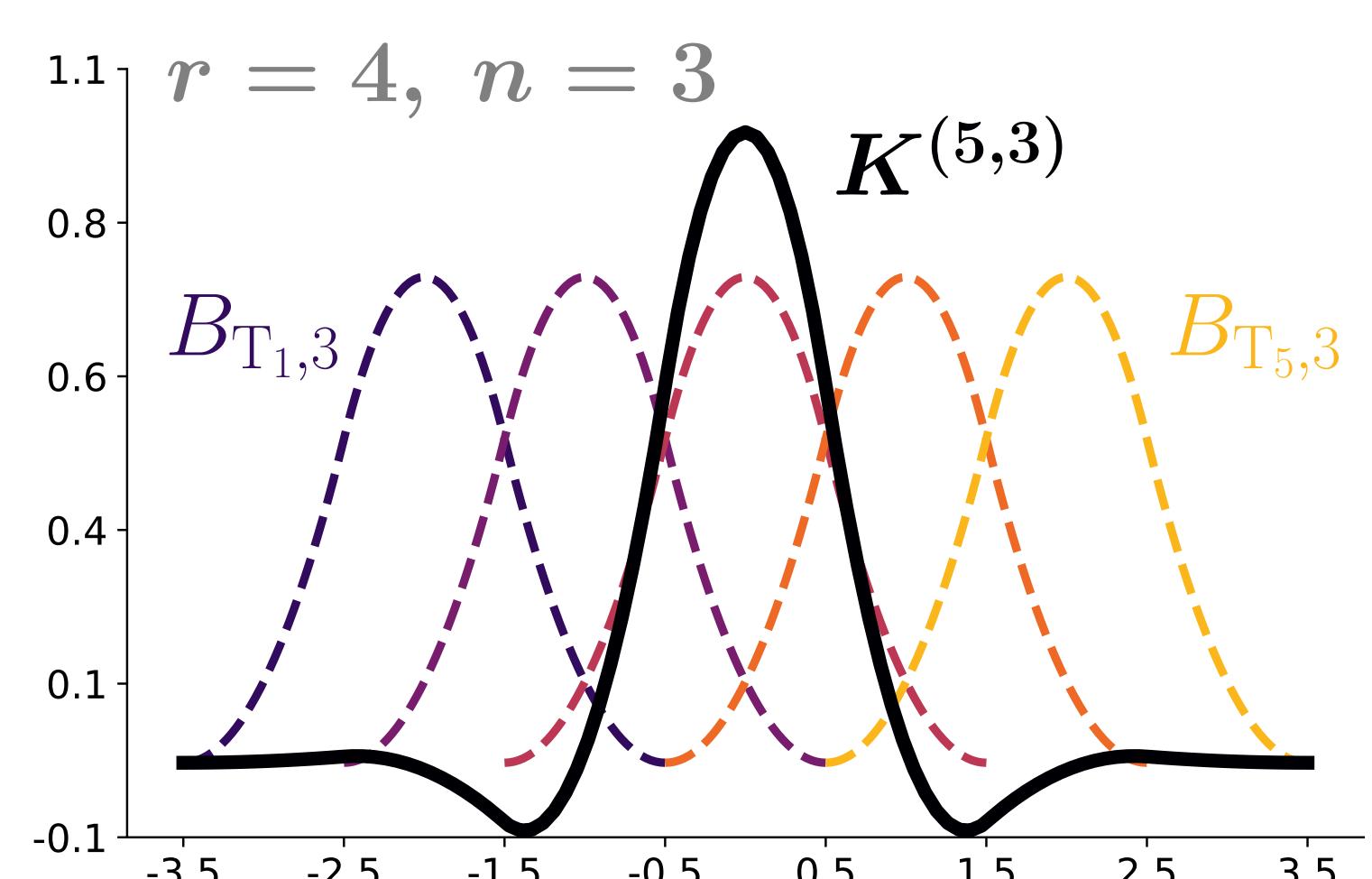
**Applications:** flow visualization, shock detection, multiresolution analysis, cut cells, ...

## SIAC Overview

Post-process data  $u$ :  $u^* = \mathbf{K} \star u$

$$\text{SIAC Kernel: } \mathbf{K}^{(r+1,n)}(\cdot) = \sum_{\gamma=1}^{r+1} c_\gamma \mathbf{B}_{T_\gamma, n}(\cdot)$$

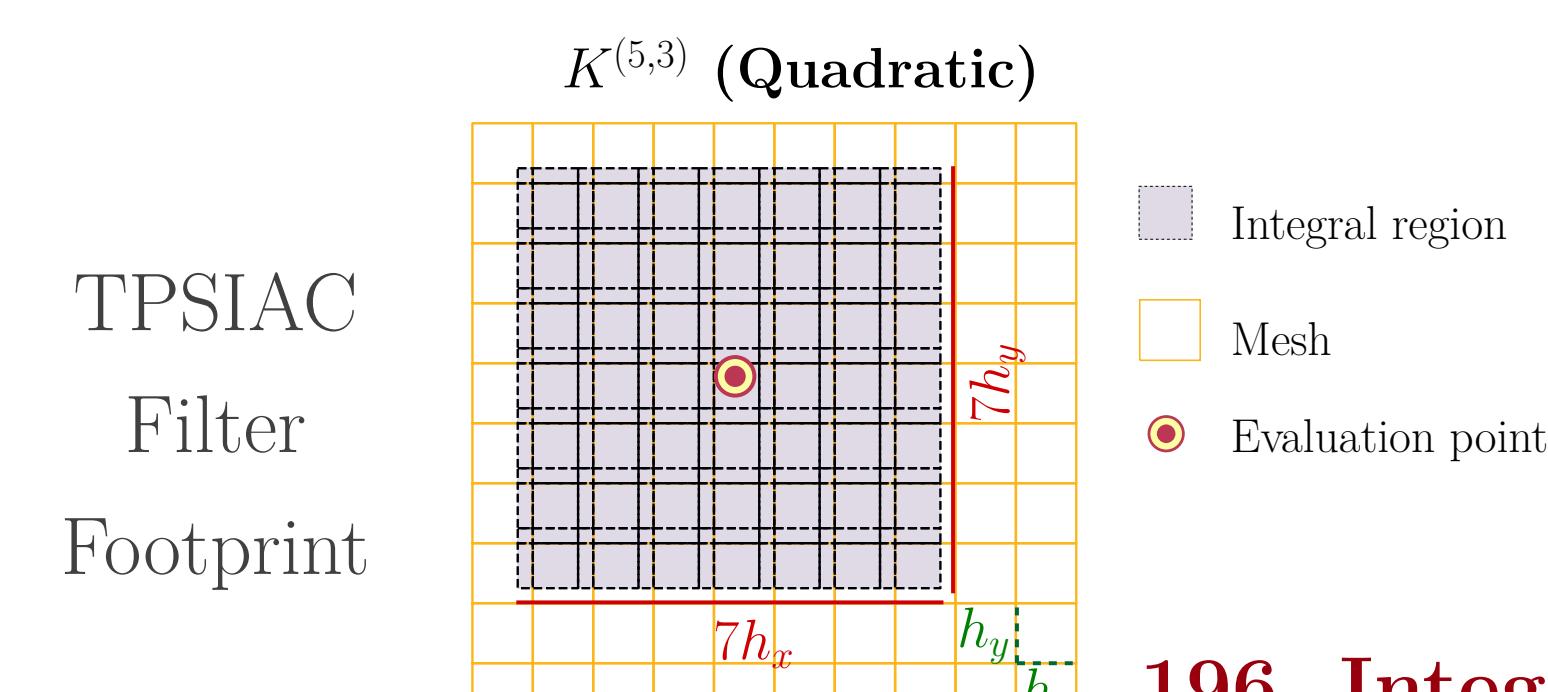
$c_\gamma$  kernel weights — maintain  $r$  moments  
 $B_{T_\gamma, n}$   $n^{\text{th}}$ -order central B-spline  
 $T$  knot sequence,  $n-2$  smoothness



## Tensor Product vs Line Filter in 2D

**Tensor Product (TPSIAC):**  $\mathbf{K} = \mathbf{k}_x \otimes \mathbf{k}_y$

$$u^*(\bar{x}, \bar{y}) = \frac{1}{h_x h_y} \int_{\mathbb{R}} \int k_x(\bar{x}-x) k_y(\bar{y}-y) u(x, y) dy dx$$

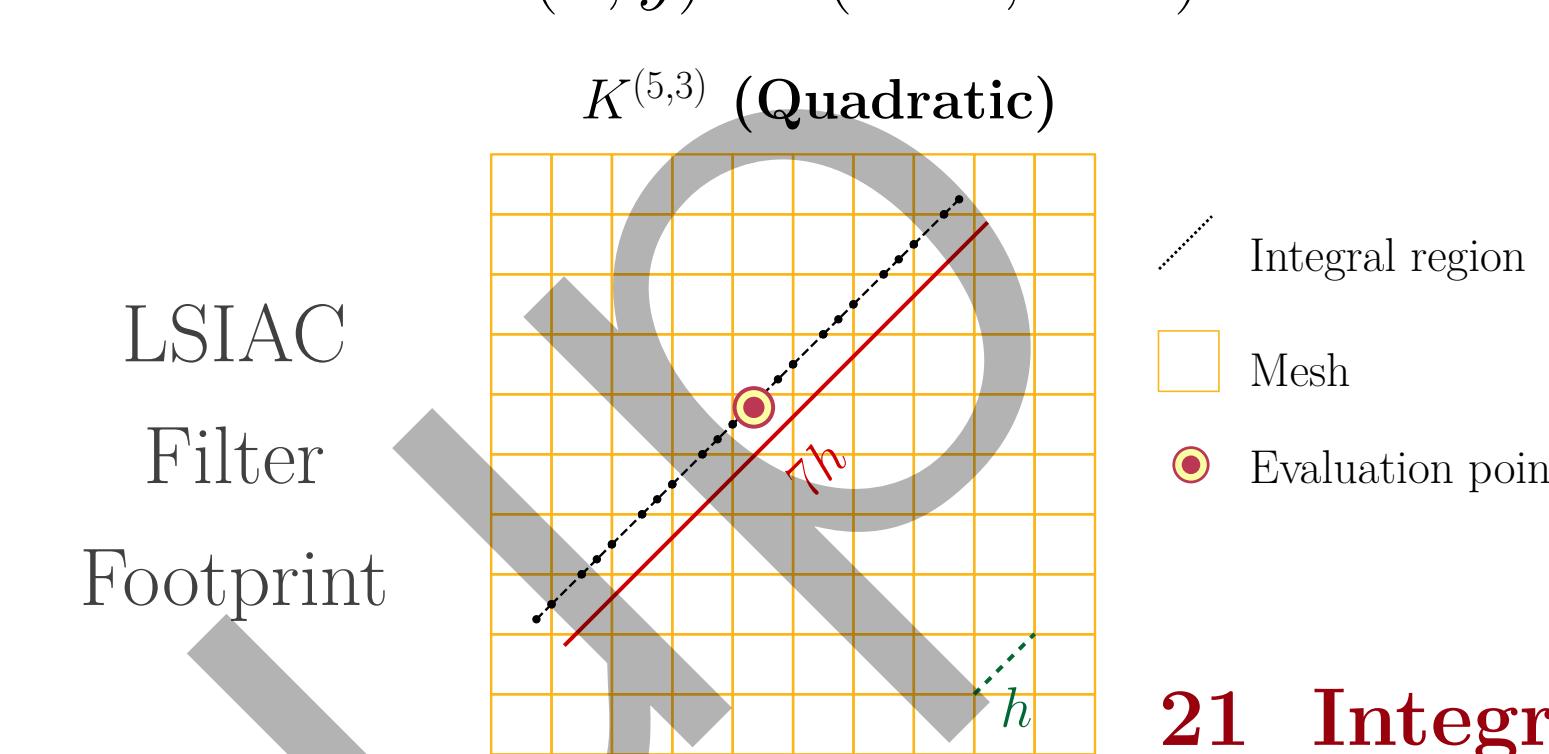


196 Integrals

**Line (LSIAC):**  $\mathbf{K} = \mathbf{k}_\Gamma$

$$u^*(\bar{x}, \bar{y}) = \frac{1}{h} \int_{\mathbb{R}} k_\Gamma(t) u(\Gamma(t)) dt$$

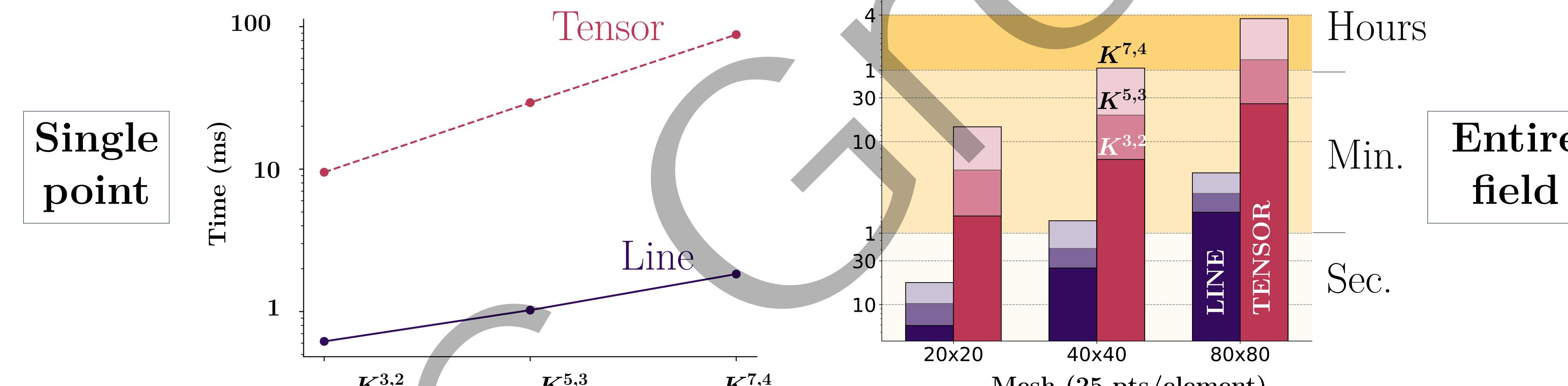
$$\Gamma = (\bar{x}, \bar{y}) + t(\cos \theta, \sin \theta)$$



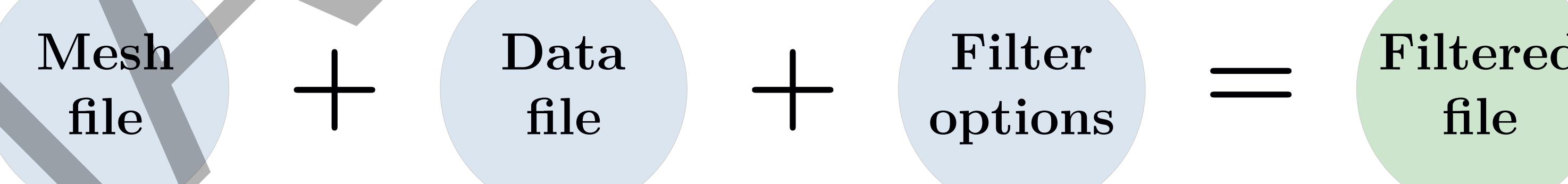
21 Integrals

**Computation:** split integral based on elements and spline breaks

**Filter Performance:** CPU time comparison



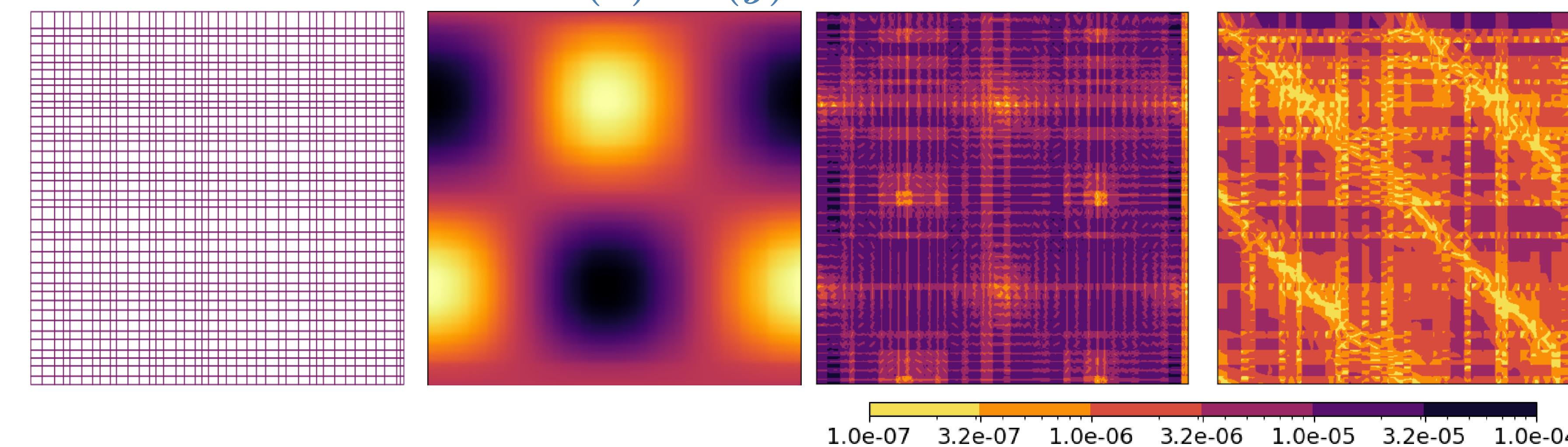
**Software Package in Julia**



## Numerical Results (Quads, Nonuniform Mesh)

### Solution

Mesh:  $40 \times 40$   $u = \sin(x) \cos(y)$



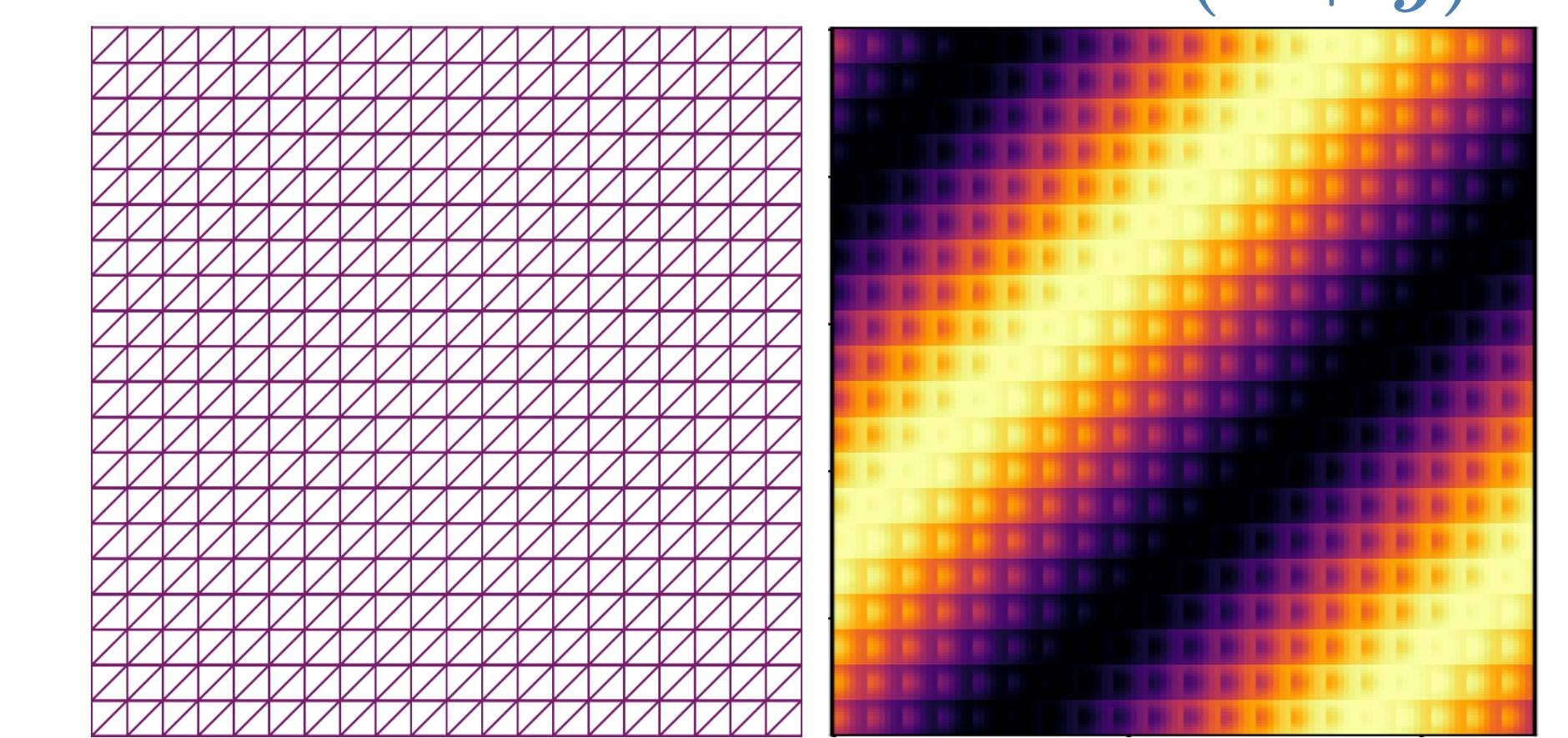
### Error contours (log)

DG      LSIAC

## Numerical Results (Triangles)

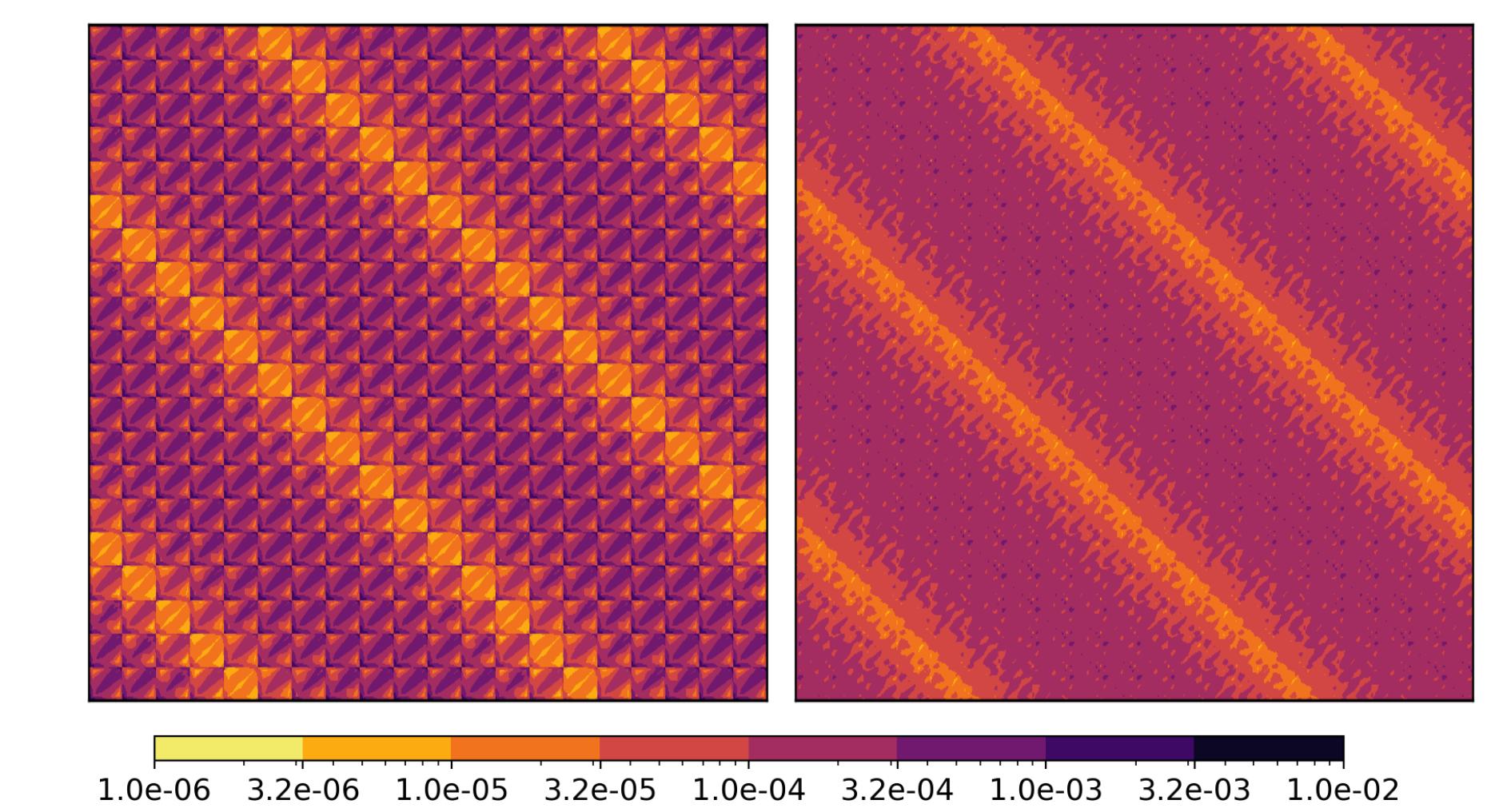
### Solution

Mesh:  $40 \times 20$   $u = \sin(x + y)$



### Error Contours (log)

DG      LSIAC



## Conclusion

- The SIAC filter achieves the optimal superconvergence rate  $2p+1$ , reduces error, and removes oscillations.
- LSIAC is more computationally efficient than the TPSIAC filter and is implemented in our Julia Software Package.

## References

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- J. Docampo-Sánchez, J. K. Ryan, M. Mirzargar, and R. M. Kirby. Multi-dimensional filtering: Reducing the dimension through rotation. *SIAM Journal on Scientific Computing*, 39(5):A2179–A2200, 2017.

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